

C A L C U L A T O R

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HP-67/97

Quickly Now, Where Does Waveguide Cutoff Occur?

Key in the dimensions and take a stab at the cutoff frequency to simplify ridged- and finned-waveguide designs. The calculator provides the cutoff wavelength and characteristic impedance.

The programs presented here use a standard Hewlett-Packard Pac routine called "Calculus and Roots of $f(x)$," to compute the cutoff wavelengths of all TE_{m0} modes in single- and double-ridged waveguides. They also determine the fundamental mode characteristic impedances of the guides at an infinite frequency. The standard routine to calculate the roots of the transverse resonance conditions occupies the first 112 steps in both programs.

These programs have the features described in the standard Pac manual. That is, the designer can opt for the pause function to see the routine converge. The value of the resonance condition, as well as the first derivative, can be obtained for any input value of λ_c , the cutoff frequency. This latter feature is helpful in the search for roots and their identification. Typically, the accuracy of these pro-

grams is ± 2 percent if certain restrictions, that will be outlined later, are followed.

Understand the key equations in waveguide design

All dimensions of the cross section of ridged waveguides are defined in Fig. 1. And, since in most applications the ridges are situated on the broad walls of the waveguide, the two lowest order modes are normally the TE_{10} and TE_{20} modes.

The cutoff wavelength of these modes satisfies what can be called the transverse resonance condition. This condition describes the standing-wave pattern in the transverse direction of the guide, and accounts for the capacitive step discontinuities introduced by the ridges as well as the presence of the sidewalls. The equivalent transverse network of the structure is shown in Fig. 2.

If the m of a given TE_{m0} mode is odd, the electric field is at its maximum value in the center of the cross section; thus the transverse resonance condition is:

$$(b/d) \tan 2s\pi/\lambda_c + B_o/Y_o - \cot \pi(2a-2s)/\lambda_c = 0 \quad (1)$$

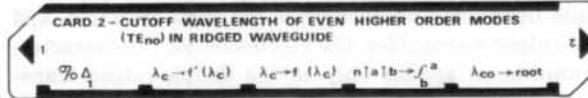
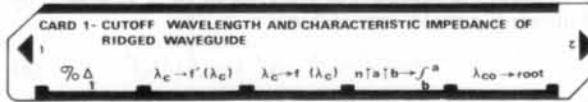
Conversely, for an even m , the electric field is zero at the

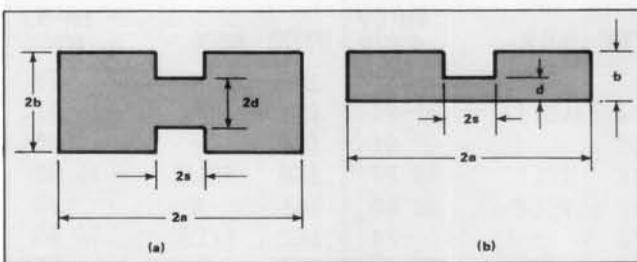
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HP STANDARD PAC "CALCULUS AND ROOTS OF $f(x)$ " ROUTINE

STEP	KEY	HP97 code	STEP	KEY	HP97 code	STEP	KEY	HP97 code	STEP	KEY	HP97 code	STEP	KEY	HP97 code
001	*LBLA	21 11	024	X#Y	-41	047	RTN	24	078	RCLB	36 00	093	RCLA	36 11
002	STOI	35 46	025	R4	-31	048	*LBLC	21 13	071	GSBi	23 45	094	RCLB	36 00
003	RTN	24	026	%	55	049	STOB	35 00	072	RCLC	36 13	095	STOA	35 11
004	*LBLB	21 16 15	027	X=0?	16-43	050	GSBi	23 45	073	ST+0	35-55 00	096	-	-45
005	F0?	16 23 00	028	LSTM	16-63	051	RTN	24	074	X	-35	097	RCLD	36 14
006	GT00	22 00	029	STOC	35 13	052	*LBLD	21 14	075	ST+9	35-55 09	098	RCLB	36 12
007	SFE	16 21 00	030	2	02	053	XZY	-41	076	RCLB	36 12	099	STOD	35 14
008	1	01	031	+	-24	054	STOB	35 00	077	X#I	16-41	100	-	-45
009	RTN	24	032	-	-45	055	-	-45	078	DSZI	16 25 46	101	+	-24
010	*LBLG	21 00	033	STOA	35 11	056	XZY	-41	079	GT07	22 07	102	X	-35
011	0	00	034	STOB	35 00	057	STOB	35 12	080	STOI	35 46	103	ST-0	35-45 00
012	CF0	16 22 00	035	GSBi	23 45	058	+	-24	081	RCL9	36 09	104	RCLB	36 00
013	RTN	24	036	STOD	35 14	059	STOC	35 13	082	RTN	24	105	F0?	16 23 00
014	*LBLd	21 16 11	037	RCLA	36 11	060	2	02	083	*LBLB	21 15	106	FSE	16 51
015	SF1	16 21 01	038	RCLC	36 13	061	+	-24	084	FIX	-11	107	+	-24
016	STOE	35 15	039	+	-55	062	ST+0	35-55 00	085	GSBB	23 12	108	RND	16 24
017	RTN	24	040	STOB	35 00	063	0	00	086	RCLB	36 12	109	X#0?	16-42
018	*LBLS	21 12	041	GSBi	23 45	064	ST09	35 09	087	GT00	22 00	110	GT06	22 06
019	EEX	-23	042	STOB	35 12	065	RCLB	36 12	088	*LBL6	21 06	111	RCLB	36 00
020	CHS	-22	043	RCLD	36 14	066	X#I	16-41	089	RCLB	36 00	112	RTW	24
021	Z	02	044	-	-45	067	*LBL7	21 07	090	GSBi	23 45			
022	RCLB	36 15	045	RCLC	36 13	068	X#I	16-41	091	STOB	35 12			
023	F1?	16 23 01	046	÷	-24	069	STOB	35 12	092	*LBL8	21 00			

PROGRAM 1				PROGRAM 2				
STEP	KEY	HP97 code	STEP	KEY	HP97 code	STEP	KEY	HP97 code
113	*LBL1	21 01	169	2	82	113	R/S	51
114	1	01	170	x	-35	114	*LBL1	21 01
115	RCL2	36 02	171	ST07	35 07	115	1	01
116	RCL0	36 00	172	RCL4	36 04	116	RCL2	36 02
117	÷	-24	173	Pi	16-24	117	RCL0	36 00
118	ST05	35 05	174	x	-35	118	÷	-24
119	X ²	53	175	RCL0	36 00	119	ST05	35 05
120	-	-45	176	÷	-24	120	X ²	53
121	JX	54	177	ST06	35 06	121	-	-45
122	1/X	52	178	TAN	43	122	JX	54
123	1	01	179	RCL2	36 02	123	1/X	52
124	-	-45	180	x	-35	124	1	01
125	ST06	35 06	181	RCL3	36 03	125	-	-45
126	RCL3	36 03	182	÷	-24	126	ST06	35 06
127	RCL2	36 02	183	+	-55	127	RCL3	36 03
128	÷	-24	184	RCL1	36 01	128	RCL2	36 02
129	Pi	16-24	185	RCL4	36 04	129	÷	-24
130	X ²	-35	186	-	-45	130	Pi	16-24
131	2	02	187	Pi	16-24	131	x	-35
132	÷	-24	188	x	-35	132	2	02
133	ST07	35 07	189	RCL0	36 00	133	÷	-24
134	COS	42	190	÷	-24	134	ST07	35 07
135	X ²	53	191	ST08	35 08	135	COS	42
136	ST08	35 08	192	TAN	43	136	X ²	53
137	X ²	53	193	1/X	52	137	ST08	35 08
138	X	-35	194	-	-45	138	X ²	53
139	RCL7	36 07	195	RTN	24	139	x	-35
140	SIN	41	196	*LBL2	21 02	140	RCL7	36 07
141	ST07	35 07	197	PRTX	-14	141	SIN	41
142	X ²	53	198	RCL6	36 06	142	ST07	35 07
143	X ²	53	199	SIN	41	143	X ²	53
144	RCL6	36 06	200	RCL2	36 02	144	X ²	53
145	X	-35	201	x	-35	145	RCL6	36 06
146	1	01	202	RCL3	36 03	146	x	-35
147	+	-55	203	÷	-24	147	1	01
148	÷	-24	204	RCL7	36 07	148	+	-55
149	RCL7	36 07	205	RCL8	36 08	149	÷	-24
150	1/X	52	206	2	02	150	RCL7	36 07
151	LN	32	207	÷	-24	151	1/X	52
152	+	-55	208	TAN	43	152	LN	32
153	RCL7	36 07	209	+	-55	153	+	-55
154	X ²	53	210	RCL6	36 06	154	RCL7	36 07
155	3	03	211	COS	42	155	X ²	53
156	X	-35	212	x	-35	156	3	03
157	1	01	213	+	-55			
158	-	-45	214	RCL5	36 05			
159	RCL5	36 05	215	÷	-24			
160	X	-35	216	3	03			
161	RCL8	36 08	217	7	07			
162	X	-35	218	7	07			
163	4	04	219	÷	-24			
164	÷	-24	220	Pi	16-24			
165	X ²	53	221	÷	-24			
166	+	-55	222	1/X	52			
167	RCL5	36 05	223	PRTX	-14			
168	X	-35	224	RTN	24			





1. Sections of double-ridged (a) and single-ridged waveguide (b) define dimensions to calculate cutoff wavelength and impedance at infinite frequency.

(text continued from p. 70)

center, and the transverse resonance condition is:

$$\left\{ \frac{(\tan 2s\pi/\lambda_c)/[b/d - (B_o/Y_o) \tan 2s\pi/\lambda_c]}{+ \tan \pi(2a-2s)/\lambda_c} = 0 \right\} \quad (2)$$

where λ_c is the cutoff wavelength and B_o/Y_o is the normalized step susceptance. It's important to note that these expressions apply to both the double- and single-ridged waveguides, provided that the a, b, d, and s dimensions are defined according to Fig. 1.

The susceptance B_o/Y_o in these equations is given by:¹

$$B_o/Y_o = 2(2b/\lambda_c) \left\{ \ln \csc(\pi d/2b) + \frac{Q \cos^4(\pi d/2b)}{1+Q \sin^4(\pi d/2b)} + \frac{1}{16}(2b/\lambda_c)^2 [1-3 \sin^2(\pi d/2b)]^2 \cos^4(\pi d/2b) \right\} \quad (3)$$

with:

$$Q = [1 - (2b/\lambda_c)^2]^{-1/2} - 1 \quad (4)$$

The cutoff wavelength of the fundamental TE_{10} mode is the first root of the condition (Eq. 1) while the next higher mode, TE_{20} , starts propagating at a wavelength that corresponds to the first root of the condition set forth in Eq. 2.

Determine cutoff wavelength; then proceed

The characteristic impedance of a ridged waveguide is defined as the ratio of the voltage across the center to the guide and the total longitudinal current flowing in its top wall.^{2,3} As soon as the cutoff wavelength of the TE_{10} mode is known, the expression for the characteristic impedance at an infinite frequency can be evaluated.

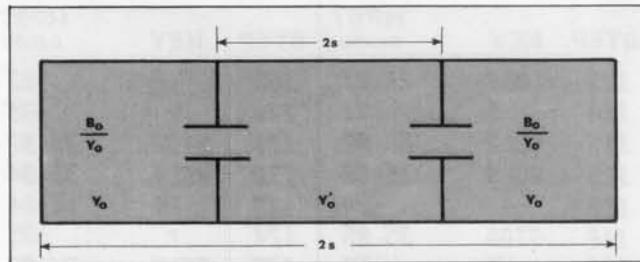
$$Z_{o\infty} (\text{ohms}) = 120\pi^2(2b/\lambda_c)/ \left\{ (b/d)\sin 2s\pi/\lambda_c + [B_o/Y_o + \tan \pi(2a-2s)/2\lambda_c] \cos 2s\pi/\lambda_c \right\} \quad (5)$$

The guided wavelength and the characteristic impedance at an arbitrary frequency, f, are related to λ_g and $Z_{o\infty}$ by the following simple expression:

$$\lambda/\lambda_g = Z_{o\infty}/Z_o = \sqrt{1 - (\lambda/\lambda_c)^2} \quad (6)$$

where λ is the free-space wavelength corresponding to the frequency f, λ_g is the guided wavelength, and Z_o is the characteristic impedance at this frequency.

While the guided wavelength is the same in single- and double-ridged waveguides, the expression for the characteristic impedance applies only to the double-ridged transmission medium. However, to obtain the characteristic impedance value for a single-ridged guide, simply divide the double-ridged value by two. Also, note that the bandwidth



2. The capacitive-step discontinuities introduced by the ridges and the presence of sidewalls are considered in this equivalent transverse network of a ridged waveguide.

of the fundamental mode is the frequency range between the cutoff frequencies of the TE_{10} and TE_{20} modes.

As mentioned earlier, there are certain restrictions associated with the programs. The equivalent transverse network depicted in Fig. 2 is valid only for $2b/\lambda_c < 1$. The expression for B_o/Y_o (Eq. 3) is accurate to within 6 percent and has a natural tendency to improve as s/b approaches zero. Additionally, a-s must be larger than b in order to avoid significant higher-order mode interaction between the ridge and the narrow walls of the guide. Within these restrictions, the cutoff wavelength is usually obtained with an accuracy of ± 2 percent.

An example to ease the pain

Suppose you're told to design a single-ridged waveguide with the following dimensions (refer to Fig. 2):

$$\begin{aligned} 2a &= 2.840 \text{ in.,} \\ b &= 0.5 \text{ in.,} \\ 2s &= 0.25 \text{ in., and} \\ d &= 0.10 \text{ in.} \end{aligned}$$

Find the guided wavelength and characteristic impedance at 2 GHz for the fundamental TE_{10} mode, and the recommended operating range for this mode of propagation.

First, the cutoff wavelength (λ_c) and the characteristic impedance at an infinite frequency ($Z_{o\infty}$) must be evaluated. To do this, the data must be input. After entering or loading program number 1 (card 1: cutoff wavelength and characteristic impedance of ridged waveguide), including the "RAD" and "DSP" mode, and selecting function number 1 (1, A), the following values are loaded into the storage registers:

$$\begin{aligned} 2a &= 2.84 \text{ into } R_1 & 2d &= 0.20 \text{ into } R_3 \\ 2b &= 1.00 \text{ into } R_2 & 2s &= 0.25 \text{ into } R_4 \end{aligned}$$

Now, a guess at the cutoff wavelength must be entered. This guess must be an educated one, close enough to the true value to insure that the program converges toward the correct value. A value of $\lambda_c = 5a = 7.1$ is close, and then pushing E prompts the calculator to search for the root. After about 50 seconds, the result is displayed as $\lambda_c = 8.645$. If there is any doubt about the order of the root obtained, a graphical solution of the transverse resonance condition can be made by drawing all three terms of Eq. 1 versus $2\pi/\lambda_c$, and taking:

$$B_o/Y_o \approx (4b/\lambda_c) \ln \frac{4b}{\pi d}$$

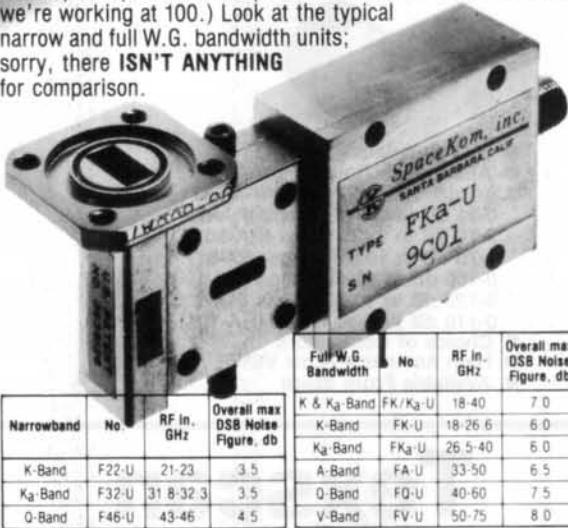
The next higher-order roots are 2.337 and 1.263, which correspond to the cutoff wavelength of the TE_{30} and TE_{50} modes, respectively. The cutoff wavelengths of these higher-order modes are obtained with less accuracy than that of

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SIMPLIFY WAVEGUIDE DESIGNS

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the fundamental mode.

Now, to obtain the characteristic impedance at an infinite frequency, press **GSB 2**. The calculator immediately prints $\lambda_c = 8.645$, and $Z_{o\infty} = 111.187$ five seconds later. And, since a single-ridged waveguide is under investigation, the impedance value must be divided by two. Therefore: $\lambda_c = 8.645 \text{ in.} = 21.959 \text{ cm}$, and $Z_{o\infty} = 55.594 \text{ ohms}$.

The cutoff frequency, $f_c = 1.366 \text{ GHz}$.

According to Eq. 6, the guided wavelength and the characteristic impedance at 2 GHz are $\lambda_g = 20.539 \text{ cm}$, and $Z_o = 76.121 \text{ ohms}$.

The recommended operating range of a waveguide usually starts at $1.25 f_{c_1}$ and ends at $0.95 f_{c_2}$ (with f_{c_1} and f_{c_2} being the cutoff frequencies of the fundamental and second order modes, respectively); f_{c_2} is calculated by finding the root of Eq. 2. To this end, load program 2 (card 2: cutoff wavelength of even higher order modes [TE_{no}] in ridged waveguide) and enter a guess of the cutoff wavelength (about 2a); after about 40 seconds, the calculator will display $\lambda_c = 2.645 \text{ in.}$, which corresponds to a cutoff frequency of 4.466 GHz for the TE_{20} mode. The recommended operating range, therefore, lies between 1.7 GHz and 4.2 GHz, which exceeds one octave.**

References

1. N. Marcuvitz, "Waveguide Handbook," *MIT Radiation Laboratory Series*, Boston Technical Publishers, Inc., Boston, MA, No. 10, (1964).
2. S. B. Cohn, "Properties Of Ridged Waveguide," *Proceedings of the IRE*, Vol. 35, No. 8, pp. 783-788, (August, 1947).
3. T. S. Chen, "Calculation of the Parameters of Ridge Waveguides," *IRE Transactions on Microwave Theory and Techniques*, Vol. MTT-5, pp. 12-17, (January, 1957).

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MICROWAVES • December 1979